

Clockwork Quantum Universe

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Abstract. Besides the purely digital or analog interpretation of reality there is a third possibility which incorporates important aspects of both. This is the cyclic formulation of elementary systems, in which elementary particles are represented as classical strings vibrating in compact space-time dimensions with periodic boundary conditions. We will address these cyclic solutions as “de Broglie internal clocks”. They constitute the deterministic gears of a consistent semi-classical description of quantum relativistic physics, providing in addition an appealing formulation of the notion of time.

INTRODUCTION

One of the possible ways to introduce the cyclic interpretation of elementary particle physics [1–9] is provided by ‘t Hooft’s determinism [10–13]. It states that “there is a close relationship between a quantum harmonic oscillator” with angular frequency $\bar{\omega} = 2\pi/T_t$, *e.g.* a single mode of an ordinary second-quantized field of energy $\bar{E} = \hbar\bar{\omega}$, “and a classical particle moving along a circle” with time periodicity T_t . By assuming the time period T_t on a lattice with N sites, it turns out that if the experimental time accuracy is too low ($\Delta t \gg T_t$), at every observation the system appears in an arbitrary discretized phase of its cyclic evolution, *i.e.* on an arbitrary site of the periodic lattice. Since the underlying periodic dynamics are too fast to be observed, the evolution has an apparent aleatoric behavior as if observing a “clock under a stroboscopic light” [12]. The evolution operator $\mathcal{U}(\Delta t = \varepsilon) = \exp[-\frac{i}{\hbar}\mathcal{H}\varepsilon]$ is given in terms of a $N \times N$ matrix and the model is analogous to a harmonic system of N masses and springs on a ring. In the limit of large N , the frequency eigenvectors $|\phi_n\rangle$ obey to the relation $\mathcal{H}|\phi_n\rangle \sim \hbar\bar{\omega}(n+1/2)|\phi_n\rangle$ which actually describes the energy eigenvalues $E_{n+1/2} = \hbar\bar{\omega}(n+1/2)$ of a quantum harmonic oscillator with periodicity T_t - apart for an “unimportant” phase in front of the operator $U(\varepsilon)$ which reproduces the factor 1/2 in the eigenvalues [13–15] and which can be regarded a twist factor on the Periodic Boundary Conditions (PBCs). The idea is that, due to the extremely fast cyclic dynamics, we loose information about the underlying classical theory and we observe a statistical theory that matches QM. For this reason we speak about deterministic or pre-quantum theories. Since the cyclic time interval T_t is supposed on a lattice, the ‘t Hooft determinism can be classified as purely digital. It is recently evolved into the idea of “classical cellular automata” [16] (*i.e.* a deterministic model with interesting correspondences between elementary particles and black holes). However, if we take the continuous limit of the ‘t Hooft deterministic model by assuming an infinite number of lattice sites $N \rightarrow \infty$, it is easy to see that the system of springs and masses turns out to be a vibrating string embedded in a cyclic time dimension, that is a bosonic classical field $\Phi(x, t)$ embedded in a compact time dimension of length T_t and PBCs. Formally, through discrete Fourier transform, to a compact variable corresponds a quantized conjugate variable, that is to say a variable which takes discrete values. Hence, a compact dimension yields to a digital description of the conjugate space. Considering the relation $\bar{E} = \hbar\bar{\omega}$, to the intrinsically periodic system with $t \in [0, T_t]$ there is associated the quantized energy spectrum $E_n = n\hbar\bar{\omega} = nh/T_t$. The energy is the digital conjugate variable of a cyclic time variable. More in general, since we experimentally observe an energy-momentum space on a lattice (quantized energy-momentum spectrum), it is natural to try to describe QM in terms of intrinsic space-time periodicities. In [1, 2] we have shown that, similarly to a particle in a box, relativistic fields can be actually quantized by imposing their characteristic de Broglie space-time periodicities as constraints.

Our assumption of dynamical periodic fields can be regarded as a combination of the Newton’s law of inertia and de Broglie hypothesis of undulatory mechanics: *elementary isolated systems must be supposed to have persistent periodicities as long as they do not interact*. Such an assumption of intrinsic periodicity is also implicit in the operative definition of time (and for some aspects in the action-reaction law). Time can only be defined by counting the number of cycles of isolated phenomena *supposed* to be periodic. For a consistent formalization of time in physics, there must be an assumption of intrinsic periodicity for free elementary systems! In modern physics a second is defined as the duration of 9,192,631,770 characteristic *cycles* of the Cs atom ($T_{Cs} \sim 10^{-10}s$). For the central role of time in physics,

the assumption of isochronism of the pendulum made by Galileo in the cathedral of Pisa can be regarded as one of the foundational acts of physics. Such an assumption of persistent periodic phenomena allowed a sufficiently accurate definition of time to study the motion of bodies and in turn the formulation of theories of dynamics. The definition of relativistic clock given by A. Einstein [17] is: “by a clock we understand anything characterized by a phenomenon passing periodically through identical phases so that we must assume, by the principle of sufficient reason, that all that happens in a given period is identical with all that happens in an arbitrary period”. The whole information of such a relativistic clock is contained in a single period. Thus, by using the terminology of extra dimensional theories, in reference clocks time can be formalized as a compact, analog dimension with PBCs. In this way every free particle, represented as a non-interacting cyclic field with intrinsic de Broglie time periodicities T_t , can be regarded of as reference clock, also known as “de Broglie internal clocks” [18, 19].

Since the measure of time is a counting process it also has a digital nature. This intrinsically leads to the Heisenberg uncertain principle, [1, 4]. In fact, in a “de Broglie clock”, to determine the energy $\bar{E} = \hbar\bar{\omega}$ with good accuracy $\Delta\bar{E}$ we must count a large number of cycles, that is to say we must observe the system for a long time Δt , according to the relation $\Delta\bar{E}\Delta t \gtrsim \hbar$. Moreover, since intrinsic periodicity means that the only possible energy eigenmodes are those with an integer number of cycles, we obtain the Bohr-Sommerfeld quantization condition (it can be shown that the periodicity condition $E_n T_t = nh$ can be more in general written as $\oint E_n dt = nh$ for interacting systems). Intrinsic periodicity can be in fact used to solve non-relativistic quantum problems [1, 4].

For a Lorentz covariant formulation of the theory we must consider that the de Broglie time periodicity induces spacial de Broglie periodicities λ^i , and that these space-time periodicities, as well as the energy-momentum quantized spectrum, transforms in a relativistic way. In other words, since $T_t = h/\bar{E}$, the de Broglie time periodicity must be regarded as dynamical. As every time interval and in analogy with the Doppler effect, T_t transforms in a relativistic way. The proper-time intrinsic periodicity T_τ fixes the upper bond of the time periodicity T_t because the mass is the lower bond of the energy. For instance, by denoting the reference system by the spatial momentum $\bar{\mathbf{p}}$, where $p_i = h/\lambda^i$, we have $T_\tau \geq T_t(\bar{\mathbf{p}})$ and $\bar{M}c^2 \leq \bar{E}(\bar{\mathbf{p}})$. The heavier the mass the faster the proper-time periodicity. Hence, even a light particle such as the electron has (in a generic reference frame) intrinsic time periodicity equal or faster than $\sim 10^{-20}s$, *i.e.* the time periodicity in a generic reference frame is always faster than its proper-time periodicity. It should be noted that such a periodicity is many orders of magnitude away from the characteristic time periodicity of the cesium atomic clock, which by definition is of the order of $10^{-10}s$, and that it is extremely fast even if compared with the present experimental resolution in time ($\sim 10^{-17}s$). Thus, for every known matter particle (with the exception of neutrinos) we are in the case of too fast periodic dynamics as in the ‘t Hooft determinism. The de Broglie intrinsic clock of elementary particles can also be imagined as a dice, named “de Broglie deterministic dice” [6], rolling with time periodicity T_t . In fact we inevitably have a too low revolution in time, so that at every observation the system appears in an aleatoric phase of its evolution. Indeed, as for a clock under a stroboscopic light or a dice rolling too fast with respect to our time resolution, we can only predict the outcomes statistically (an observer with infinite time resolution would not have fun playing dices). From the results presented in [1, 2] and summarized here we will see that such a statistical description associated to intrinsically periodic phenomena formally matches ordinary QM. We may also note that, on a cyclic geometry such as that associated with intrinsic periodicity, there exist many possible classical paths, characterized by different winding (digital) numbers, between every initial and final point. Thus the evolution of a cyclic field is described by a sum over classical cyclic paths. The result is a formal matching with the ordinary Feynman Path Integral. In this essay we will only describe some published results or announce some others that will be published soon. The reader interested in more technical details or to the mathematic proofs may refers to [1–4].

RELATIVISTIC GEARS

The relativistic generalization of Newton’s law of inertia can be stated as follows: every isolated elementary system has persistent four-momentum $\bar{p}_\mu = \{\bar{E}/c, \bar{\mathbf{p}}\}$. On the other hand, the de Broglie formulation of QM prescribes that to the four-momentum must be associated a “periodic phenomenon” of four-angular-frequency, according to the relation $\bar{\omega}_\mu = \bar{p}_\mu c/\hbar$. Here we will assume that every elementary system is described as field of intrinsic de Broglie periodicity $T^\mu = \{T_t, \bar{\lambda}_x/c\} = 2\pi/\bar{\omega}_\mu$ imposed as constraint. As the Newton’s law of inertia does not imply that every point particle moves on a straight line, our assumption of intrinsic periodicities does not mean that the physical world should appear to be periodic. In fact, the four-periodicity T^μ is fixed dynamically by the four-momentum through the de Broglie-Planck relation

$$T^\mu \bar{p}_\mu c = h. \quad (1)$$

From this follows that the variation of four-momentum occurring during interactions implies a corresponding modulation of the intrinsic periodicity of the fields. This guarantees time ordering and relativistic causality.

Similarly to the 't Hooft deterministic model, a free cyclic field $\Phi(\mathbf{x}, t)$ is a tower of frequency eigenmodes $\phi_n(x)$ with energies $E_n(\bar{\mathbf{p}}) = n\bar{E}(\bar{\mathbf{p}}) = n\hbar\bar{\omega}(\bar{\mathbf{p}})$,

$$\Phi(\mathbf{x}, t) = \sum_n A_n \phi_n(\mathbf{x}) u_n(t), \quad \text{where} \quad u_n(t) = e^{-i\omega_n(\bar{\mathbf{p}})t}. \quad (2)$$

By bearing in mind the relation $\bar{E}(\bar{\mathbf{p}}) = \hbar\bar{\omega}(\bar{\mathbf{p}})$, the quantized energy spectrum $E_n(\bar{\mathbf{p}})$ is nothing but the harmonic frequency spectrum $\omega_n(\bar{\mathbf{p}}) = n\bar{\omega}(\bar{\mathbf{p}})$ of a string vibrating with time periodicity $T_t(\bar{\mathbf{p}})$ ¹. This quantization is the field theory analogous of the semiclassical quantization of a “particle” in a box, it also shares deep analogies with the Matsubara and the Kaluza-Klein (KK) theory [20]. Since in this case the whole physical information of the system is contained in a single four-period T^μ , our intrinsically four-periodic free field can be described by a bosonic action in compact space-time dimensions with PBCs

$$\mathcal{S}_{\lambda_s} = \int_0^{T^\mu} d^4x \mathcal{L}_{\lambda_s}(\partial_\mu \Phi, \Phi). \quad (3)$$

It is important to note that PBCs minimize the action at the boundaries — in particular the ones of the time dimension. Therefore PBCs have the same formal validity of the usual (Synchronous) BCs assumed in ordinary field theory. In this way all the symmetries of the relativistic bosonic theory are preserved as usual. In particular it guarantees that the theory is Lorentz invariant. In fact we may consider a generic global Lorentz transformation

$$dx^\mu \rightarrow dx'^\mu = \Lambda^\mu_\nu dx^\nu, \quad \bar{p}_\mu \rightarrow \bar{p}'_\mu = \Lambda^\nu_\mu \bar{p}_\nu. \quad (4)$$

The phase of the field is invariant under four-periodic T^μ translations $\exp[-ix^\mu \bar{p}_\mu] = \exp[-i(x^\mu + cT^\mu) \bar{p}_\mu]$. and it is a scalar quantity under Lorentz transformations - de Broglie phase harmony. In this way we find that, as every generic space-time interval, the four-periodicity is actually a contravariant four-vector

$$T^\mu \rightarrow T'^\mu = \Lambda^\mu_\nu T^\nu. \quad (5)$$

The four periodicity T^μ can be thought of as describing a reciprocal energy-momentum lattice $p_{n\mu} = n\bar{p}_\mu$. Invariance can also be inferred by noticing that after the transformation of variables (4), the integration region of the free action (3) turns out to be transformed as well,

$$\mathcal{S}_{\lambda_s} = \int_0^{T'^\mu} d^4x' \mathcal{L}_{\lambda_s}(\partial'_\mu \Phi', \Phi'). \quad (6)$$

Therefore, in the new reference system, the new four-periodicity T'^μ of the field is actually given by (5). That is (6) describes a system with four-momentum \bar{p}'_μ (4).

The underlying Minkowski metric induces the following constraint on the dynamical four-periodicities

$$\frac{1}{T_\tau^2} \equiv \frac{1}{T_\mu} \frac{1}{T^\mu}$$

which, considering the above de Broglie-Planck relation, is nothing but the relativistic constraint $\bar{M}^2 c^2 = \bar{p}^\mu \bar{p}_\mu$.

The resulting compact 4D formulation reproduces, after normal ordering, exactly the same quantized energy spectrum of ordinary second quantized fields. In fact, a cyclic field with mass \bar{M} turns out to have energy spectrum

$$E_n(\bar{\mathbf{p}}) = n\bar{E}(\bar{\mathbf{p}}) = n\sqrt{\bar{\mathbf{p}}^2 c^2 + \bar{M}^2 c^4}$$

of ordinary quantum field theory. Furthermore, it is easy to see that in the rest frame ($\bar{\mathbf{p}} \equiv 0$) this quantized energy spectrum is *dual* to the KK mass tower $M_n = E_n(0)/c^2 = n\bar{M}$. Indeed, for such a massive field, the assumption of periodicity along the time dimension means that in the rest frame the proper-time τ there has intrinsic periodicity

$$T_\tau = T_t(0) = \frac{h}{\bar{M}c^2}$$

¹ The theory can be regarded as a particular kind of string theory in which the compact world-line parameter plays the role of the compact world-sheet parameter.

The invariant mass \tilde{M} is not a parameter of the Lagrangian. It is fixed geometrically by the reciprocal of the proper-time intrinsic periodicity T_τ thought PBCs, and thus by the compactification lengths of the theory. In other words, by imposing intrinsic time periodicity, the world-line parameter $s = c\tau$ turns out to be compact with PBCs. It behaves similarly to the XD of a KK field with zero 5D mass and with fundamental mass \tilde{M} . As a consequence the world-line compactification length $\lambda_s = cT_\tau$ is the Compton wavelength of the field. In order to bear in mind these analogies with an XD field theory we say that the world-line parameter plays the role of a *Virtual* XD (VXD) with compactification length λ_s , [3]. It is interesting to note that, originally, T. Kaluza introduced the XD formalism as a “mathematical trick” and not as a *real* XD [21].

QUANTUM GEARS

Here we show that our cyclic description of elementary particles has a remarkable formal matching to the canonical (axiomatic) formulation of QM as well as to the Feynman Path Integral (FPI) formulation. The evolution along the compact time dimension is described by the so called bulk equations of motion $(\partial_t^2 + \omega_n^2)\phi_n(x, t) = 0$ - for the sake of simplicity in this section we assume a single spatial dimension x . Thus the time evolution of the energy eigenmodes can be written as first order differential equations $i\hbar\partial_t\phi_n(x, t) = E_n\phi_n(x, t)$. The periodic field (2) is a sum of on-shell standing waves. Actually this is the typical case where a Hilbert space can be defined. In fact, the energy eigenmodes form a complete set with respect to the inner product

$$\langle\phi|\chi\rangle \equiv \int_0^{\lambda_x} \frac{dx}{\lambda_x} \phi^*(x)\chi(x). \quad (7)$$

Therefore the energy eigenmodes define Hilbert eigenstates $\langle x|\phi_n\rangle \equiv \phi_n(x)/\sqrt{\lambda_x}$. On this base we can formally build a Hamiltonian operator $\mathcal{H}|\phi_n\rangle \equiv \hbar\omega_n|\phi_n\rangle$ and a momentum operator $\mathcal{P}|\phi_n\rangle \equiv -\hbar k_n|\phi_n\rangle$, where $k_n = n\tilde{k} = nh/\lambda_x$. Thus the time evolution of a generic state $|\phi(0)\rangle \equiv \sum_n a_n|\phi_n\rangle$ is described by the familiar Schrödinger equation

$$i\hbar\partial_t|\phi(t)\rangle = \mathcal{H}|\phi(t)\rangle. \quad (8)$$

Moreover the time evolution is given by the usual time evolution operator $\mathcal{U}(t'; t) = \exp[-\frac{i}{\hbar}\mathcal{H}(t-t')]$ which turns out to be a Marcovian (unitary) operator: $\mathcal{U}(t''; t') = \prod_{m=0}^{N-1} \mathcal{U}(t' + t_{m+1}; t' + t_m - \varepsilon)$ where $N\varepsilon = t'' - t'$.

From the fact that the spatial coordinate is in this theory a cyclic variable; by using the definition of the expectation value of an observable $\hbar\partial_x F(x)$ between two generic initial and final states $|\phi_i\rangle$ and $|\phi_f\rangle$ of this Hilbert space; and integrating by parts (7), we find

$$\langle\phi_f|\hbar\partial_x F(x)|\phi_i\rangle = i\langle\phi_f|\mathcal{P}F(x) - F(x)\mathcal{P}|\phi_i\rangle. \quad (9)$$

By assuming that the observable is such that $F(x) = x$ [22] we obtain the usual commutation relation of ordinary QM: $[x, \mathcal{P}] = i\hbar$ — or more in general $[F(x), \mathcal{P}] = i\hbar\partial_x F(x)$. The commutations relations are implicit in the assumption of intrinsic periodicity. With this result we have checked the correspondence with canonical QM.

Similarly, it is possible to prove the correspondence with the FPI formulation. In fact, it is sufficient to plug the completeness relation of the energy eigenmodes in between the elementary time evolutions of the Marcovian operator. With this elements at hand and proceeding in a complete standard way we find that the evolution of the cyclic fields turns out to be described by the usual FPI which, in phase space ($V_x = N\lambda_x$ with $N \in \mathbb{N}$ large in case of interaction, see [1, 2]), can be written as

$$\mathcal{Z} = \lim_{N \rightarrow \infty} \int_0^{V_x} \left(\prod_{m=1}^{N-1} \frac{dx_m}{V_x} \right) \prod_{m=0}^{N-1} \langle\phi| e^{-\frac{i}{\hbar}(\mathcal{H}\Delta\varepsilon_m - \mathcal{P}\Delta x_m)} |\phi\rangle, \quad (10)$$

This important result has been obtained without any further assumption than PBCs and has a simple classical interpretation. In a cyclic geometry there is an infinite set of possible classical paths with different winding numbers that link every given initial and final points. If we imagine to open this cyclic geometry we obtain a lattice with period T^μ of initial and final points linked by classical paths. The FPI obtained in (10) means that a cyclic field can self-interfere and its classical evolution is described by summing over the possible paths with different winding number. These path play the role of the non-classical paths of Feynman interpretation of QM, though they are classical paths minimizing the action in compact space-time dimensions. This means that in this path integral formulation it is not necessary to relax the classical variational principle to have self-interference.

The non-quantum limit of a massive field, *i.e.* the non-relativistic single particle description, is obtained by putting the mass to infinity so that, as shown in [1, 4], in such an effective classical limit, only the first level of the energy spectrum must be considered, [1, 4]. This leads to a consistent interpretation of the wave-particle duality and of the double slit experiment. The quantities describing only the first energy level are addressed by the bar sign. For instance, the Lagrangian of the transformed fundamental mode $\bar{\Phi}'(x')$ is $\mathcal{L}_{\lambda_s}(\partial_\mu \bar{\Phi}'(x'), \bar{\Phi}'(x'))$. Note that the transformed fundamental mode $\bar{\Phi}'(x')$ coincides with the mode of Klein-Gordon field with energy \bar{E}' and mass \bar{M} . Therefore it can be always quantized through second quantization. For this reason the analysis of the geometrodynamics of the de Broglie periodicities that we will perform below can be extended to ordinary field theory, [2]. On the other hand a massless field has infinite Compton wavelength and thus an infinite proper-time periodicity. Its quantum limit is at high frequency. In this limit the PBCs are important and we have a discretized energy spectrum, in agreement with the ordinary description of the black-body radiation (no UV catastrophe). The opposite limit described by a continuous energy spectrum is when time periodicity tends to infinity.

In the original 't Hooft model the period T_i was assumed to be of the order of the Planck time, in an attempt to avoid hidden variables, [23]. Furthermore the Hamiltonian operator was not positive defined. In our case, the intrinsic periodicities are reproduced by compact dimensions with PBCs. Therefore we have the remarkable property that QM emerges without involving any hidden-variable. The theory can in principle violates the Bell's inequality and we can actually speak about determinism. Moreover, similarly to the KK theory in which there are no tachyons, a cyclic field can have positive or negative frequency eigenmodes but the energy spectrum describes always positive energies and the Hamiltonian operator is positive defined.

GEOMETRODYNAMICS

To introduce interactions we must bear in mind that the four-periodicity T^μ is dynamically related to the four-momentum \bar{p}_μ according to the de Broglie-Planck relation (1). An isolated elementary system (*i.e.* free field) has persistent four-momentum, whereas an elementary system under a generic interaction scheme can be described in terms of corresponding variations of four-momentum along its evolution with respect to the free case

$$\bar{p}_\mu \rightarrow \bar{p}'_\mu(x) = e^a_\mu(x) \bar{p}_a. \quad (11)$$

The tetrad (or virebein) $e^a_\mu(x)$ turns out to encode the interaction scheme. But in undulatory mechanics the interaction (11) can be equivalently described in terms of corresponding local modulations of space-time periodicity

$$T^\mu \rightarrow T'^\mu(x) \sim e^\mu_a(x) T^a. \quad (12)$$

In our formalism these modulations correspond to local deformations of the compactification lengths. Roughly speaking, interactions can be formalized as local stretching of the compact dimensions of the theory. Therefore the interaction (11) turns out to be equivalently encoded in a corresponding curved space-time background, which in the limit of weak interaction can be approximated as

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) \sim e^a_\mu(x) e^b_\nu(x) \eta_{ab}. \quad (13)$$

This result can be checked by considering the transformation of space-time variables

$$dx_\mu \rightarrow dx'_\mu(x) \sim e^a_\mu(x) dx_a. \quad (14)$$

Under the approximation of weak interaction we are assuming that the T^μ transforms as an infinitesimal interval dx^μ . After this transformation of variables (diffeomorphism) with determinant of the Jacobian $\sqrt{-g(x)}$, the free action (3) turns out to be²

$$\mathcal{S}_{\lambda_s} \sim \int e^\mu_a(x) T^a d^4x' \sqrt{-g} \mathcal{L}_{\lambda_s}(e^a_\mu \partial_a \Phi', \Phi'). \quad (15)$$

Indeed, the transformed periodic field $\Phi'(x')$ which minimizes this action has four-periodicity T'^μ , (12), or equivalently has four-momentum \bar{p}'_μ , (11). We conclude that a field under the interaction scheme (11) is described by the solutions of the bulk equations of motion on the deformed compact background (13) and compactification lengths (12).

² For the sake of simplicity, we work in the approximation in which the compactification length can be identified with the de Broglie periodicity, [2]. A more exact description would involve Christoffel symbols.

Our geometrodynamical approach to interactions is interesting because it mimics very closely the usual geometrodynamical approach of GR. In fact, it is important to note that gravitational interaction can be interpreted as modulations of periodicity of reference clocks. For instance we may consider a weak Newton potential $V(\mathbf{x}) = -GM_\odot/|\mathbf{x}| \ll 1$. The energy on a gravitational well varies (with respect to the free case) as $\bar{E} \rightarrow \bar{E}' \sim (1 + GM_\odot/|\mathbf{x}|)\bar{E}$. According to (12) or (1), this means that the de Broglie clocks in a gravitational well are slower with respect to the free clocks $T_i \rightarrow T_i' \sim (1 - GM_\odot/|\mathbf{x}|)T_i$. Thus we have a gravitational redshift $\bar{\omega} \rightarrow \bar{\omega}' \sim (1 + GM_\odot/|\mathbf{x}|)\bar{\omega}$. With our simple schematization of interactions we have retrieved two important predictions of GR. Besides this we must also consider the analogous variation of spatial momentum and the corresponding modulation of spatial periodicities [24]. According to (13) the weak newtonian interaction turns out to be encoded in the usual linearized Schwarzschild metric

$$ds^2 \sim \left(1 - \frac{GM_\odot}{|\mathbf{x}|}\right) dt^2 - \left(1 + \frac{GM_\odot}{|\mathbf{x}|}\right) d|\mathbf{x}|^2 - |\mathbf{x}|^2 d\Omega^2. \quad (16)$$

We have found that the geometrodynamical approach to interactions actually can be used to describes linearized gravity and that the geometrodynamics of the compact space-time dimensions correspond to the usual relativistic ones.

As well known, see for instance [24], it is possible to retrieve ordinary GR from a linear formulation by including self-interactions. More naively, as we will show in detail in forthcoming papers, we may regard the metric $g_{\mu\nu}$ a dynamical field of the theory. This corresponds to introduce “by hand” a kinetic term (with appropriate coupling $16\pi G_N$) to the Lagrangian in curve space-time (11). Moreover, in order to neglect quantum corrections, we may replace the Lagrangian $\sqrt{-g}\mathcal{L}_{\lambda_s}$ of (17) with its non-quantum limit $\sqrt{-g}\bar{\mathcal{L}}_{\lambda_s}$ (*i.e.* the Lagrangian of the fundamental mode Φ). Thus we obtain the familiar Hilbert-Einstein Lagrangian

$$\bar{\mathcal{L}}_{HE} = \sqrt{-g} \left[-\frac{g^{\mu\nu}\mathcal{R}_{\mu\nu}}{16\pi G_N} + \bar{\mathcal{L}}_{\lambda_s}(e_\mu^a \partial_a \bar{\Phi}', \bar{\Phi}') \right]. \quad (17)$$

This naive procedure is similar to the derivation of the kinematic term $F_{\mu\nu}F^{\mu\nu}/-4e^2$ in gauge field theories³. Because of its geometrical meaning, the Ricci tensor is the correct mathematical object to describes the dynamical variations of the space-time compactification lengths at different interaction points. This can be intuitively seen by using the de Broglie harmony (1) and that encodes the content of four-momentum in different space-time points. Here we also mention that it is not uniquely defined “what is fixed at the boundary of the action principle of GR”, [25]. As well known, Einstein’s equation can be obtained from different action formulations, differing by boundary terms. In particular in the ordinary formulation of GR we can neglect issues related to the variations of the boundary terms and obtain Einstein equation $\mathcal{R}^{\mu\nu} = -8\pi G_N \mathcal{T}^{\mu\nu}$ as the variation of the metric directly from the Lagrangian (17). With these simple heuristic arguments we have shown that field theory in compact space-time is compatible with GR.

In forthcoming papers⁴, we will show that, by writing (11) as a minimal substitution, such a description of interactions can also be used to describe ordinary gauge interactions in terms of space-time geometrodynamics. Gauge fields turn out to encode modulations of periodicities associated to local transformations of reference frame. The assumption of PBCs at the geometrodynamical boundary of the theory lead, through a generalization of the demonstration of a generalization of (10), to the ordinary FPI of scalar QED, [2]. Such a semi-classical description of QED yield, for instance, a intuitive description of superconductivity [2, 4, 9].

We have also mentioned that a cyclic field turns out to be dual an XD field, [3, 4]. In fact the compact world-line parameter of the theory plays the role of a VXD. On the other hand we have shown that a cyclic field reproduces ordinary quantum dynamics [1, 2]. From the dualism to XD theories and from the geometrodynamical approach to interactions described above, we find that the classical evolution of the periodic fields along a deformed VXD background reproduces the quantum behavior of the corresponding interaction scheme. Thus relativistic field theory in compact 4D provides the possibility of an intuitive, unconventional interpretation of the Maldacena conjecture. In fact, according to Witten, in AdS/CFT “quantum phenomena [...] are encoded in classical geometry”. We will apply this idea to a simple Bjorken Hydrodynamical Model for Quark-Gluon-Plasma (QGP) logarithmic freeze-out [26]. In first approximation the energy momentum of the QGP can be supposed to decay exponentially (similarly to the Newton’s law of cooling for a thermodynamic system [27]). This interaction scheme is therefore described by the conformal warped tetrad $e_\mu^a = \delta_\mu^a e^{-ks}$, where s is the proper time, *i.e.* by a *virtual* AdS metric. As a result, the classical configurations of cyclic fields in such a deformed background reproduces basic, phenomenological aspects of AdS/QCD, [3].

³ In [2] we have also shown that it can be derived directly by applying the variational principle at the boundary of the theory.

⁴ One of these papers is now published in *Annals of Physics*, [2].

CONCLUSIONS

The formalism of field theory in compact space-time dimensions provides, through PBCs, a fully consistent, natural description of both the digital aspect of reality arising QM and the analog aspect of reality typical of relativity [1–9]. It must be noticed that (general and special) relativity sets the differential structure of space-time without giving any particular prescription for the BCs. On the other hand, the BCs have played an important role since the earliest days of QM (for instance as in the Bohr atom or as in the particle in a box). We have seen that relativity is compatible with compact (analog) space-time dimensions, as long as the periodicities (*i.e.* the compactification lengths) are allowed to transform in a covariant way. On the other hand, through discrete Fourier transform to a compact dimension there is naturally associated a discretized frequency spectrum, and thus a quantized energy spectrum.

The physical assumption of intrinsic periodicity leads to a formal correspondence with ordinary (axiomatic) relativistic QM in both the canonical and the Feynman formulations, as well as for many non trivial quantum phenomena, [1–9]. Time periodicity can be used to describe naturally the transition between the classical and the quantum regime of an elementary systems. We may consider for instance, the different components of an electromagnetic field in a Black-Body radiation, *i.e.* in the case of massless cyclic phenomena. The IR components correspond to the limit of nearly infinite time periodicity, *i.e.* low energy or frequency. In this case the PBCs can be neglected and the elementary system is described by fields with approximatively continuous energy spectrum (in this purely analog limit the thermal noise destroys the intrinsic periodicity in a sort of decoherence). In the limit of small time periodicity, corresponding to the UV components of a Black Body radiation, the field description of the theory is gradually replaced by digital corpuscular aspects. The PBCs can not be neglected and the energy spectrum turns out to be quantized (the thermal noise is not sufficient to destroy the intrinsic periodicity). The initial and final configurations of a periodic field form a periodic space-time lattice, so that the cyclic evolution is described by a sum over classical paths with different winding numbers (an electron does 10^{10} cycles for every “tick” of the Cs clock).

These intrinsic time periodicities of elementary particles can be identified with the so call “de Broglie internal clocks” or “de Broglie periodic phenomena” at the base of the undulatory description of modern relativistic QM. Similarly to a stopwatch, every moment in time is determined by the combination of the phases or the “ticks” of periodic cycles (*e.g.* years, months, days, hours, minutes and seconds). That is, every value of our external temporal axes (defined with reference to the digital “ticks” of the Cs-133 atomic clock) is characterized by a unique combination of the “ticks” of all the “de Broglie internal clocks”, *i.e.* elementary particles, constituting the system under investigation. In this scenario the local nature of relativistic time turns out to be enforced. The long temporal (and spatial) scales are provided by massless fields with low frequencies (the long compactifications lengths provides the underlying spatial and temporal structure of the system of elementary particles). As we have seen, it must be considered that the such clocks can modulate periodicity through interactions (exchange of energy) and that the periodicities depend dynamically on reference systems according to the relativistic laws (*e.g.* relativistic Doppler effect). This also means that the combination of two or more de Broglie clocks, *i.e.* a non elementary system of particles, leads to very chaotic evolutions as interaction is turned on. Such a description of the flow of time depends only on the reciprocal combinations and variations of the “ticks” of the de Broglie clocks. It is therefore reference frame depended in a relativistic way. Moreover the arms of the de Broglie internal clocks can be conventionally supposed to rotate clockwise or anticlockwise (but an inversion of a single clock corresponds to transform the particle to the corresponding antiparticle or *vice versa*). These further conceptual aspects are particularly interesting for the problem of the time arrow in physics.

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